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Factorization of Regge Slopes for Ordinary and New Hadrons and their Spectroscopy

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ABSTRACT

We discuss in detail a factorization property of Regge slopes between ordinary and new hadrons proposed earlier. This is supported experimentally in a known case, and the (tensor) D^{**} mass in this scheme is predicted to be (2.4 ± 0.04)GeV. In addition, we obtain relations between boson slopes and baryon slopes with four flavors. By combining these with the slope factorization property, we can then actually predict all of the baryon slopes. Predictions for both the boson and baryon spectra are also given.

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I. INTRODUCTION

Recent discoveries of new hadrons[1] strongly suggest a new degree of freedom (charm[2, 3]) in particle physics. This new degree of freedom also leads to the prediction of a rich spectrum of additional new hadrons. Hence we need a dynamics for these new hadrons. It has been known for some time that the concept of duality for hadrons in the sense of finite-energy sum rules (FESR) has convincing experimental support.[4] Before charm was found, it turned out to be very useful to combine the quark model with duality in order to construct a consistent hadron dynamics. Since charm has now been found, there is a great need for a new hadron dynamics to accommodate charm in the duality scheme of hadrons.

There are many similarities as well as differences between the ordinary and the new hadrons. One of the differences is that the Regge slopes of the new hadrons appear to be smaller than those of the ordinary ones if one approximates the $\psi - \chi$ trajectory by a straight line passing through the $\psi(3098)$ and the $J^P = 2^+$ $\chi(3550)$, which are exchange degenerate each other. As is well known, one can write a dual amplitude explicitly when the slopes of the Regge trajectories are equal in both the s- and the t-channels. When the two slopes are different, however, the B_4 amplitude of the Veneziano type would lead to an increasing exponential behavior as s approaches infinity at fixed angles even in the physical regions.[5] This might throw some doubt on the possibility that the new hadrons could be related to ordinary hadrons through duality.

This difficulty could be overcome in the following way.[6] Callan et al.[7], and Schnitzer and Kang[8] have suggested an interesting possibility some time ago, in order to understand the difference in the slopes of ordinary and new hadrons in the resonance regions. They took a string picture of hadrons[9] in which a quark and anti-quark with finite masses are attached by a massless string, and considered the rigid rotation of such a system. This would give rise to the leading Regge trajectory if quantized. If one considers the $\psi - \chi$ Regge trajectory as the $c\bar{c}$ system, for example, the slope of the $\psi - \chi$ trajectory is small for low spins since one can make the nonrelativistic approximation while it would approach a universal slope at high spins since the c-quark mass would be negligible compared with the energy of the string in that region. Thus, the slopes of the new and ordinary hadrons will both approach a universal slope at infinity even though these slopes are different in the resonance regions (see Fig. 1). If this is the case, the new and ordinary hadrons could be related through duality without contradicting Mandelstam's arguments[5].

It is not yet known, however, how to construct an explicit dual model for arbitrary trajectories in practice even if it is possible in principle. Suppose we can approximate trajectories by straight lines at low and moderately high energies and consider the FESR duality connecting a sum of s-channel resonances to the t-channel Regge poles in these regions. A B_4 amplitude connecting any pair of trajectories in the s- and t- channels satisfies this duality. Expanding such an amplitude at moderately high energies and imposing the factorization property of Regge residues[10] we were led to a new factorization property[11, 12] of Regge slopes relating ordinary and new bosons and also one[11] relating ordinary and new baryons.

This paper is an expanded version of Ref. 11, and in addition includes new predictions and applications. By requiring a definite signature for the u-channel Δ in the (u, t) and (u, s) dual terms for $\pi N \to \pi N$ as well as for all possible combinations of external particles with four flavors, we obtain relations between boson slopes and baryon slopes. Thus we can calculate the actual magnitudes of all baryon slopes by combining the slope factorization property with the above relations connecting baryon slopes with boson slopes. We then obtain rich predictions for the baryon spectrum as well as the boson spectrum.

In Sec. II, we give a derivation of our factorization property for boson slopes. We also show that the factorization property continues to hold even in the presence of a finite number of satellite terms. In Sec. III, we derive the factorization of baryon slopes. We show that the results still hold even when parity doublets in the resonance regions are eliminated by a finite number of satellite terms. In Sec. IV, we derive relations between boson slopes and baryon slopes. Sec. V is devoted to experimental comparisons. We test our predictions for boson slopes in a known case. We also calculate the magnitude of the slope for each boson and baryon spectra. In Sec. VI, we give concluding remarks and discussions.

II. FACTORIZATION OF REGGE SLOPES--BOSONS

For simplicity we first consider meson-meson scattering in order to derive a factorization property of Regge slopes relating ordinary and new bosons.

Suppose we take the following reactions

$$D\bar{D} \to \rho \to D\bar{D}, \quad D\bar{D} \to \rho \to \pi\pi, \quad \pi\pi \to \rho \to \pi\pi$$

in the t-channel. Then the t-channel and ρ-pole residues have to factorize[10]

$$\beta_{D\bar{D}\rho D\bar{D}}(t) \cdot \beta_{\pi\pi\rho\pi\pi}(t) = \left[\beta_{D\bar{D}\rho\pi\pi}(t)\right]^2 . \tag{2.1}$$

Let us choose the $\pi^+\pi^-$ scattering amplitude to be

$$A^{\pi^+\pi^-}(s,\ t) = -\lambda B_4(\alpha_\rho(s),\ \alpha_\rho(t)) , \tag{2.2}$$

where $\alpha_\rho(t)$ denotes the ρ - f trajectory function and $B_4(x,\ y) \equiv \frac{\Gamma(1 - x)\Gamma(1 - y)}{\Gamma(1 - x - y)}$.

Then we obtain

$$\beta_{\pi\pi\rho\pi\pi}(t) = \frac{\lambda\pi}{\Gamma(\alpha_\rho(t))} \alpha_\rho'^{\ \alpha_\rho(t)} . \tag{2.3}$$

Similar considerations for the $D\bar{D}$ and πD scattering amplitudes[6] immediately lead to[13]

$$\beta_{D\bar{D}\rho D\bar{D}}(t) \sim \frac{\pi}{\Gamma(\alpha_\rho(t))} \alpha_\psi'^{\ \alpha_\rho(t)} \tag{2.4}$$

$$\beta_{D\bar{D}\rho\pi\pi}(t) \sim \frac{\pi}{\Gamma(\alpha_\rho(t))} \alpha'_{D^*}{}^{\alpha_\rho(t)} \tag{2.5}$$

up to a constant factor. Here α'_ψ (α'_{D^*}) denotes the slope of the ψ - χ (D^* - D^{**}) trajectory, which is assumed to be linear up to moderately high energies (Fig. 1). Substitution of Eqs. (2.3), (2.4) and (2.5) into Eq. (2.1) leads us to

$$\frac{(\alpha'_\psi)^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))} \cdot \frac{(\alpha'_\rho)^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))} = \left[\frac{(\alpha'_{D^*})^{\alpha_\rho(t)}}{\Gamma(\alpha_\rho(t))}\right]^2 \tag{2.6}$$

which must hold for any value of t. Therefore, we must have the relation

$$\alpha'_\psi \alpha'_\rho = (\alpha'_{D^*})^2 \quad . \tag{2.7}$$

It is to be noted here that constant factors in front of the B_4-function also have to factorize.

Moreover, we can show that this relation continues to hold even if the amplitudes include a finite number of satellite terms.

Let us write the $\pi^+\pi^-$, $D^+\pi^-$ and D^+D^- scattering amplitudes as

$$A^{\pi^+\pi^-}\left(\alpha_\rho(s),\ \alpha_\rho(t)\right) = - \sum_{m=1,2,\ldots}^{N} \sum_{n \geq m}^{N} \lambda^{\pi\pi}_{nm} \frac{\Gamma\left(n - \alpha_\rho(s)\right)\Gamma\left(m - \alpha_\rho(t)\right)}{\Gamma\left(n - \alpha_\rho(s) - \alpha_\rho(t)\right)} , \tag{2.8}$$

$$A^{D^+\pi^-}\left(\alpha_{D^*}(s),\ \alpha_\rho(t)\right) = - \sum_{m=1,2,\ldots}^{N} \sum_{n \geq m}^{N} \lambda^{\pi D}_{nm} \frac{\Gamma\left(n - \alpha_{D^*}(s)\right)\Gamma\left(m - \alpha_\rho(t)\right)}{\Gamma\left(n - \alpha_{D^*}(s) - \alpha_\rho(t)\right)} , \tag{2.9}$$

$$A^{D^+D^-}\left(\alpha_\psi(s),\ \alpha_\rho(t)\right) = -\sum_{m=1,2,\ldots}^{N}\ \sum_{n\geq m}^{N} \lambda_{nm}^{D\bar{D}} \frac{\Gamma\left(n-\alpha_\psi(s)\right)\Gamma\left(m-\alpha_\rho(t)\right)}{\Gamma\left(n-\alpha_\psi(s)-\alpha_\rho(t)\right)} \quad . \tag{2.10}$$

Each term in the sums contributes to the leading behaviors in s at high s.[14] Considerations similar to those we had before immediately lead us to

$$\beta_{\pi\pi\rho\pi\pi}(t) \simeq \pi \sum_{m}^{N}\ \sum_{n\geq m}^{N} \frac{\lambda_{nm}^{\pi\pi}(-1)^{m-1}}{\Gamma\left(1-m+\alpha_\rho(t)\right)} \cdot \alpha_\rho'^{\ \alpha_\rho(t)} , \tag{2.11}$$

$$\beta_{D\bar{D}\rho\pi\pi}(t) \simeq \pi \sum_{m}^{N}\ \sum_{n\geq m}^{N} \frac{\lambda_{nm}^{\pi D}(-1)^{m-1}}{\Gamma\left(1-m+\alpha_\rho(t)\right)} \cdot \alpha_{D^*}'^{\ \alpha_\rho(t)} , \tag{2.12}$$

$$\beta_{D\bar{D}\rho D\bar{D}}(t) \simeq \pi \sum_{m}^{N}\ \sum_{n\geq m}^{N} \frac{\lambda_{nm}^{D\bar{D}}(-1)^{m-1}}{\Gamma\left(1-m+\alpha_\rho(t)\right)} \cdot \alpha_\psi'^{\ \alpha_\rho(t)} . \tag{2.13}$$

Substituting these into Eq. (2.1), we are led to

$$(\alpha_\psi' \alpha_\rho')^{\alpha_\rho(t)} = (\alpha_{D^*}'^2)^{\alpha_\rho(t)} , \tag{2.14}$$

as well as a similar factorization for the remaining part. It is to be noted here that these two relations must hold separately because of their different functional forms. Hence we again obtain Eq. (2.7) this time from Eq. (2.14).

Using the ρ-residue factorization for the reactions $D\bar{D} \to \rho \to D\bar{D}$, $D\bar{D} \to \rho \to K\bar{K}$ and $K\bar{K} \to \rho \to K\bar{K}$ as a second example, we obtain another new relation

$$\alpha_{\psi}' \cdot \alpha_{\phi}' = (\alpha_{F^*}')^2 \quad . \tag{2.15}$$

Similar considerations for $K\bar{K} \to \rho \to K\bar{K}$, $K\bar{K} \to \rho \to \pi\pi$ and $\pi\pi \to \rho \to \pi\pi$ lead to the relation

$$\alpha_{\phi}' \cdot \alpha_{\rho}' = (\alpha_{K^*}')^2 \quad . \tag{2.16}$$

The ϕ-residue factorization for $F\bar{F} \to \phi \to F\bar{F}$, $F\bar{F} \to \phi \to K\bar{K}$ and $K\bar{K} \to \phi \to K\bar{K}$ does not lead to a new relation but reduces to Eq. (2.7). Similarly, the ψ factorization through $F\bar{F} \to \psi \to F\bar{F}$, $F\bar{F} \to \psi \to D\bar{D}$, $D\bar{D} \to \psi \to D\bar{D}$ reduces to Eq. (2.16). Thus we have three independent relations without any inconsistency.

III. FACTORIZATION OF REGGE SLOPES--BARYONS

We shall now turn to a discussion of meson-baryon scattering with four flavors. Suppose we can write the (s, u) dual term for the invariant amplitudes $A^{(I)}(s, u)$ and $B^{(I)}(s, u)$.[15, 4] (Here, the superscript I denotes an isospin index in the u-channel.) The parity-conserving helicity amplitudes free from kinematical singularities in the u-channel are then expressed in terms of $A^{(I)}$ and $B^{(I)}$ as follows:

$$\tilde{F}^{(I)\pm}(\sqrt{u}, s) = \mp\left(A^{(I)} + MB^{(I)}\right) - \sqrt{u}\, B^{(I)} \quad . \tag{3.1}$$

Here $\tilde{F}^{(I)+}(\sqrt{u}, s) = \left((E_u - M)/8\pi\sqrt{u}\right) f_2{}^I(\sqrt{u}, s)$, $\tilde{F}^{(I)-} = \left((E_u + M)/8\pi\sqrt{u}\right) f_1{}^I(\sqrt{u}, s)$ and $\tilde{F}^{(I)\pm}(\sqrt{u}, s)$ includes only the $\tau P = \pm$ u-channel Regge exchanges in the leading order in s. Discussions similar to those in the previous section immediately lead to the factorization properties for baryon-Regge slopes.[11]

Here we start from the general case in which both the $A^{(I)}$ and $B^{(I)}$ include a finite number of satellite terms. The dual terms for the $A^{(I)}$ function which give the leading Regge behavior $s^{\alpha_B(u) - \frac{1}{2}}$ for large s at fixed u can be written as

$$A^{(I)} = - \sum_{q=0,1,\ldots}^{N} \sum_{p \geq q}^{N} \lambda_{pq}^{A(I)} \frac{\Gamma\left(p - \bar{\alpha}_{B1}(s)\right)\Gamma\left(q - \bar{\alpha}_B(u)\right)}{\Gamma\left(p - \bar{\alpha}_{B1}(s) - \bar{\alpha}_B(u)\right)} \tag{3.2}$$

where $\bar{\alpha}_B = \alpha_B - \frac{1}{2}$, and α_B is a baryon trajectory function. Similarly, we can write

$$B^{(I)} = - \sum_{q=0,1,\ldots}^{N} \sum_{p \geq q}^{N} \lambda_{pq}^{B(I)} \frac{\Gamma\left(p - \bar{\alpha}_{B1}(s)\right)\Gamma\left(q - \bar{\alpha}_B(u)\right)}{\Gamma\left(p - \bar{\alpha}_{B1}(s) - \bar{\alpha}_B(u)\right)} \quad . \tag{3.3}$$

Substitution of Eqs. (3.2) and (3.3) into Eq. (3.1) gives

$$\tilde{F}^{(I)-} = - \sum_{q=0,1,\ldots}^{N} \sum_{p \geq q}^{N} \left\{ \lambda_{pq}^{A(I)} - (\sqrt{u} - M)\lambda_{pq}^{B(I)} \right\} \frac{\Gamma\left(p - \bar{\alpha}_{B1}(s)\right)\Gamma\left(q - \bar{\alpha}_B(u)\right)}{\Gamma\left(p - \bar{\alpha}_{B1}(s) - \bar{\alpha}_B(u)\right)} . \tag{3.4}$$

Expanding the imaginary part of Eq. (3.4) at moderately high energies in the sense of Ref. 13, we have

$$\mathrm{Im}\tilde{F}^{(I)-} \xrightarrow[s;\ \mathrm{large}]{} \gamma^{I}(\sqrt{u})\alpha_{B1}'^{\,\bar{\alpha}_B(u)}\, s^{\bar{\alpha}_B(u)} \tag{3.5}$$

with

$$\gamma^{(I)}(\sqrt{u}) = \pi \sum_{q}^{N} \sum_{p \geq q}^{N} \left\{ \lambda_{pq}^{A(I)} - (\sqrt{u} - M)\lambda_{pq}^{B(I)} \right\} \frac{(-1)^{p-1}}{\Gamma\left(1 - p + \bar{\alpha}_B(u)\right)} . \tag{3.6}$$

Denoting the $\tilde{F}^{(I)-}$ for $a + b \to B \to d + c$ in the u-channel as $\tilde{F}^{(I)-}(ab \to B \to dc)$, we can write

$$\mathrm{Im}\tilde{F}^{(I)-}(ab \to B \to dc) \xrightarrow[s;\ \mathrm{large}]{} \gamma^{(I)}(\sqrt{u})_{ab \to B \to dc}\, \alpha'_{b\bar{c}}{}^{\bar{\alpha}_B(u)}\, s^{\bar{\alpha}_B(u)} . \tag{3.7}$$

Here B1 was replaced by $b\bar{c}$ since α_{B1} is a baryon trajectory in the $b\bar{c}$ channel.

Suppose we take the reactions $ab \to B \to ba$, $ab \to B \to dc$ and $cd \to B \to dc$ in the u-channel (Fig. 2). Then the u-channel B-pole residues have to factorize[10]

$$\gamma^{(I)}(\sqrt{u})_{ab \to B \to ba}\, \alpha'_{b\bar{a}}{}^{\bar{\alpha}_B(u)} \cdot \gamma^{(I)}(\sqrt{u})_{cd \to B \to dc}\, \alpha'_{d\bar{c}}{}^{\bar{\alpha}_B(u)}$$

$$= \left\{ \gamma^{(I)}(\sqrt{u})_{ab \to B \to dc}\, \alpha'_{b\bar{c}}{}^{\bar{\alpha}_B(u)} \right\}^2 . \tag{3.8}$$

Since the functions $\gamma^{(I)}(\sqrt{u})_{ab \to B \to dc}$ and $\alpha'_{b\bar{c}}{}^{\bar{\alpha}_B(u)}$ are completely different, the following two relations have to hold separately:

$$\gamma^{(I)}(\sqrt{u})_{ab \to B \to ba} \cdot \gamma^{(I)}(\sqrt{u})_{cd \to B \to dc} = \left(\gamma^{(I)}(\sqrt{u})_{ab \to B \to dc}\right)^2 \quad . \tag{3.9}$$

$$\alpha_{b\bar{a}}{}'^{\,\bar{\alpha}_B(u)} \; \alpha_{d\bar{c}}{}'^{\,\bar{\alpha}_B(u)} = \left(\alpha_{b\bar{c}}{}'^{\,\bar{\alpha}_B(u)}\right)^2 \quad . \tag{3.10}$$

Therefore, we are led to

$$\alpha_{b\bar{a}}' \, \alpha_{d\bar{c}}' = (\alpha_{b\bar{c}}')^2 \quad . \tag{3.11}$$

It is to be noted here that this relation still holds even when parity doublets in the resonance regions are eliminated by a finite number of satellite terms.

Let us consider each of the cases which arise when $\tau P = -$ and the u-channel B baryons are composed of four flavors:

a. Δ exchange

Suppose we take the reactions $\pi N \to \Delta \to N\pi$, $\pi N \to \Delta \to \Sigma K$, $K\Sigma \to \Delta \to \Sigma K$ in the u-channels. Then Eq. (3.11) gives us a relation

$$\alpha_\Delta' \cdot \alpha_{\Xi^*}' = (\alpha_{Y^*}')^2 \tag{3.12}$$

between s-channel baryon slopes. As we shall see later (Sec. IV), this kind of relation applies both to the octet and the decouplet. Similar considerations for $\pi N \to \Delta \to N\pi$, $\pi N \to \Delta \to C\bar{D}$, $\bar{D}C \to \Delta \to C\bar{D}$ immediately lead to

$$\alpha_\Delta' \cdot \alpha_{X^*}' = (\alpha_{C^*}')^2 \quad . \tag{3.13}$$

b. Y^* exchange

We obtain

$$\alpha_{Y^*}' \cdot \alpha_{\Omega}' = (\alpha_{\Xi^*}')^2 \tag{3.14}$$

from $\pi\Sigma \to Y^* \to \Sigma\pi$, $\pi\Sigma \to Y^* \to \Xi K$, $K\Xi \to Y^* \to \Xi K$, and

$$\alpha_{Y^*}' \cdot \alpha_{X_s^*}' = (\alpha_{S^*}')^2 \tag{3.15}$$

from $\pi\Sigma \to Y^* \to \Sigma\pi$, $\pi\Sigma \to Y^* \to S\bar{D}$, $\bar{D}S \to Y^* \to S\bar{D}$.

c. C^* exchange

Here we have

$$\alpha_{C^*}' \cdot \alpha_{T^*}' = (\alpha_{S^*}')^2 \tag{3.16}$$

from $\pi C \to C^* \to C\pi$, $\pi C \to C^* \to SK$, $KS \to C^* \to SK$, and

$$\alpha_{C^*}' \cdot \alpha_{\Theta}' = (\alpha_{X^*}')^2 \tag{3.17}$$

from $\pi C \to C^* \to C\pi$, $\pi C \to C^* \to X\bar{D}$, $\bar{D}X \to C^* \to X\bar{D}$.

d. Ξ^* exchange

The Ξ^*-residue factorization for $\bar{K}\Sigma \to \Xi^* \to \Sigma K$, $\bar{K}\Sigma \to \Xi^* \to S\bar{F}$ and $\bar{F}S \to \Xi^* \to S\bar{F}$ does not lead to any new relation but leads to Eq. (3.13).

e. X^* exchange

Similarly, the X^* factorization for $DC \to X^* \to CD$, $DC \to X^* \to SK$ and $KS \to X^* \to SK$ leads to Eq. (3.12).

f. S^* exchange

There is no new relation.

g. Ω exchange

The Ω factorization for $\overline{K}\Xi \rightarrow \Omega \rightarrow \Xi\overline{K}$, $\overline{K}\Xi \rightarrow \Omega \rightarrow T\overline{F}$ and $\overline{F}T \rightarrow \Omega \rightarrow T\overline{F}$ gives us Eq. (3.15).

h. Θ exchange

The factorization for $DX \rightarrow \Theta \rightarrow XD$, $DX \rightarrow \Theta \rightarrow X_s F$ and $FX_s \rightarrow \Theta \rightarrow X_s F$ leads to Eq. (3.16).

i. T^* exchange

Eq. (3.17) is obtained through $\overline{K}S \rightarrow T^* \rightarrow S\overline{K}$, $\overline{K}S \rightarrow T^* \rightarrow X_s\overline{F}$, $\overline{F}X_s \rightarrow T^* \rightarrow X_s \overline{F}$ factorization.

j. X_s^* exchange

Similarly, Eq. (3.14) is obtained through $DS \rightarrow X_s^* \rightarrow SD$, $DS \rightarrow X_s^* \rightarrow TF$, $FT \rightarrow X_s^* \rightarrow TF$ factorization.

Thus the above baryon exchanges with all possible combinations of external particles lead us to a set of results where in each case one of the six relations (3.12), (3.13), (3.14), (3.15), (3.16), (3.17) holds between u-channel baryon slopes. If we twist the final states, we obtain relations between t-channel boson slopes. In each of the cases mentioned above, we also obtain one of the three relations (2.7), (2.15), (2.16) derived for meson-meson scattering in Sec. II.

It is interesting to notice that these six relations manifest the factorization of quark numbers for each flavor, i.e., we have the following kind of relations[17] for baryon slopes with four flavors:

$$\left(u^{p_1}s^{q_1}c^{r_1}\right)\left(u^{p_3}s^{q_3}c^{r_3}\right) = \left(u^{p_2}s^{q_2}c^{r_2}\right) \quad . \tag{3.18}$$

We may visualize the various baryon slopes in Fig. 3 as a consequence of Eqs. (3.12) theough (3.17). At this stage, however, the magnitudes of the baryon slopes have not been predicted. We shall show in Sec. IV that one can relate baryon slopes with boson slopes and so one can predict each baryon slope by combining these with the slope factorization property.

IV. RELATIONS BETWEEN BOSON SLOPES AND BARYON SLOPES

We require a definite signature for the u-channel Δ trajectory in the (u, t) and (u, s) dual terms for πN scattering since the Δ has a definite signature.[19] A (u, t) term of the form

$$B_4\left(\overline{\alpha}_\Delta(u),\ \alpha_\rho(t)\right) \equiv \frac{\Gamma\left(1 - \overline{\alpha}_\Delta(u)\right)\Gamma\left(1 - \alpha_\rho(t)\right)}{\Gamma\left(1 - \overline{\alpha}_\Delta(u) - \alpha_\rho(t)\right)} \tag{4.1}$$

which behaves as $\Gamma\left(1 - \overline{\alpha}_\Delta(u)\right)\alpha_\rho'^{\,\overline{\alpha}_\Delta(u)}\, s^{\overline{\alpha}_\Delta(u)}$ for large s[13] at fixed u, and a (u, s) term of the form

$$B_4\left(\overline{\alpha}_\Delta(u),\ \overline{\alpha}_\Delta(s)\right) \tag{4.2}$$

which behaves as $\Gamma\left(1-\overline{\alpha}_\Delta(u)\right)\alpha_\Delta'^{\,\overline{\alpha}_\Delta(u)}\, s^{\overline{\alpha}_\Delta(u)}$ for large s at fixed u, generate a negative signature if $\alpha_\rho'^{\,\overline{\alpha}_\Delta(u)} = \alpha_\Delta'^{\,\overline{\alpha}_\Delta(u)}$ for all values of u. Therefore, we must have the relation[18]

$$\alpha_\rho' = \alpha_\Delta' \quad . \tag{4.3}$$

It is to be noted here that generally there exists a (u, s) term of the form $B\left(\overline{\alpha}_\Delta(u),\ \overline{\alpha}_{N^*}(s)\right)$ ($N^* = N_{\alpha-\gamma}$) in addition to the $B\left(\overline{\alpha}_\Delta(u),\ \overline{\alpha}_\Delta(s)\right)$. We will show the following relation to hold:

$$\alpha_\rho' = \alpha_\Delta' = \alpha_{N^*}' \quad . \tag{4.4}$$

Suppose we consider a combination of terms

$$B_4\left(\overline{\alpha}_\Delta(u),\ \alpha_\rho(t)\right) - \left[aB_4\left(\overline{\alpha}_\Delta(u),\ \overline{\alpha}_\Delta(s)\right) + bB_4\left(\overline{\alpha}_\Delta(u),\ \overline{\alpha}_{N^*}(s)\right)\right]$$
$$\xrightarrow[\text{s; large}]{} \Gamma\left(1-\overline{\alpha}_\Delta(u)\right)s^{\overline{\alpha}_\Delta(u)}\left[\alpha_\rho'^{\,\overline{\alpha}_\Delta(u)} - e^{-i\pi\overline{\alpha}_\Delta(u)}\left\{a\alpha_\Delta'^{\,\overline{\alpha}_\Delta(u)} + b\alpha_{N^*}'^{\,\overline{\alpha}_\Delta(u)}\right\}\right] , \tag{4.5}$$

with a ≠ 0, b ≠ 0. In order to have a negative signature for the Δ, we must have

$$1 = a\left(\frac{\alpha_\Delta'}{\alpha_\rho'}\right)^{\overline{\alpha}_\Delta(u)} + b\left(\frac{\alpha_{N^*}'}{\alpha_\rho'}\right)^{\overline{\alpha}_\Delta(u)} \quad . \tag{4.6}$$

If one assumes $\alpha_\Delta'/\alpha_\rho' \neq \alpha_{N^*}'/\alpha_\rho'$, one can easily arrive at a contradiction. On the other hand, if one assumes $\alpha_\Delta'/\alpha_\rho' \neq \alpha_{N^*}'/\alpha_\rho'$, one can easily show that this quantity has to be equal to unity. Thus, we arrive at Eq. (4.4). As long as the quark number for each flavor in any baryon is the same, this equality has to hold.

Similar considerations with all possible combinations of external particles with four flavors lead to the following relations: By requiring a definite signature for the u-channel Δ in the $B_4\left(\bar{\alpha}_\Delta(u),\ \alpha_{K^*}(t)\right)$ and $B_4\left(\bar{\alpha}_\Delta(u),\ \bar{\alpha}_{Y^*}(s)\right)$ terms for $\pi N \rightarrow K\Sigma$, we obtain

$$\alpha_{K^*}' = \alpha_{Y^*}' \quad . \tag{4.7}$$

A similar requirement for the Δ in the $B_4\left(\bar{\alpha}_\Delta(u),\ \alpha_\phi(t)\right)$ and $B_4\left(\bar{\alpha}_\Delta(u),\ \bar{\alpha}_{\Xi^*}(s)\right)$ terms for $K\Sigma \rightarrow K\Sigma$, leads to

$$\alpha_\phi' = \alpha_{\Xi^*}' \quad . \tag{4.8}$$

Similar considerations for the Δ in the $B_4\left(\bar{\alpha}_\Delta(u),\ \alpha_{D^*}(t)\right)$ and $B_4\left(\bar{\alpha}_\Delta(u),\ \bar{\alpha}_{C^*}(s)\right)$ term for $\pi N \rightarrow \bar{D}C$ give us

$$\alpha_{D^*}' = \alpha_{C^*}' \quad . \tag{4.9}$$

We also obtain

$$\alpha_{F^*}' = \alpha_{S^*}' \tag{4.10}$$

from the $B_4\left(\bar{\alpha}_\Delta(u),\ \alpha_{F^*}(t)\right)$ and $B_4\left(\bar{\alpha}_\Delta(u),\ \bar{\alpha}_{S^*}(s)\right)$ terms for $K\Sigma \rightarrow \bar{D}C$, and

$$\alpha_\psi' = \alpha_{X^*}' \tag{4.11}$$

from the $B_4\left(\bar{\alpha}_\Delta(u),\ \alpha_\psi(t)\right)$ and $B_4\left(\bar{\alpha}_\Lambda(u),\ \bar{\alpha}_{X^*}(s)\right)$ for $\bar{D}C \to \bar{D}C$.

The other baryon slopes $\alpha_\Omega{}'$, $\alpha_{X_s^*}{}'$, $\alpha_{T^*}{}'$, $\alpha_\Theta{}'$, which are not directly related to boson slopes, can be calculated using Eqs. (3.14), (3.15), (3.16), (3.17),respectively. Hence, by combining all of the above properties, we can calculate the actual magnitudes of all the baryon slopes from boson slopes.

V. PREDICTIONS AND THE NEW HADRON SPECTROSCOPY

A. Bosons

We first test our predictions (2.7), (2.15) and (2.16) for boson slopes. We approximate trajectories such as $\psi - \chi$,[20] $\phi - f'$, $K^* - K^{**}$ in terms of straight lines passing through the corresponding pair of resonances, and the ρ trajectory by connecting the ρ and g mesons. Then we have the following values for these boson slopes $\left(\text{in } (\text{GeV}/c)^{-2}\right)$ using masses from the Particle Data Group:[21]

$$\begin{aligned} \alpha_\rho{}' &= 0.88 \\ \alpha_{K^*}{}' &= 0.82 \\ \alpha_\phi{}' &= 0.79 \\ \alpha_\psi{}' &= 0.33 \end{aligned} \qquad (5.1)$$

The left-hand side of Eq. (2.16) is evaluated to be $(0.79)(0.88) = (0.83)^2$. Thus, Eq. (2.16) is well supported experimentally within the experimental errors. The relations (2.7) and (2.15) are difficult to test directly at present. Assuming Eq. (2.7) to hold and using the values in Eq. (5.1), one can deduce

$$\alpha_{D^*}' = 0.54 \quad . \tag{5.2}$$

If we assume $\alpha_{D^*}(s)$ to be linear and $M(D^*) = 2.01$ GeV,[1] we can predict the (tensor) D^{**} mass to be 2.43 GeV (see also Ref. 22). The relation (2.15) with the values of α_ψ' and α_ϕ' given in Eq. (5.1) predicts

$$\alpha_{F^*}' = 0.51 \quad . \tag{5.3}$$

We show the ρ, K^*, ϕ, D^*, F^*, ψ trajectories as functions of $M^2 - M_V^2$ in Fig. 4.

We now discuss the spectroscopy of the new bosons. Since the boson masses predicted in the present scheme are different from those in the conventional scheme, which assumes a universal slope, one can discriminate the two alternatives by experiment. Using the boson slopes given in Eq. (5.1) and (5.2) and taking masses[21] from experiment, we obtain the following trajectories:

$$
\begin{aligned}
\alpha_\rho(s) &= 0.47 + 0.88\ s \\
\alpha_{K^*}(s) &= 0.34 + 0.83\ s \\
\alpha_\phi(s) &= 0.18 + 0.79\ s \\
\alpha_{D^*}(s) &= -1.18 + 0.54\ s \\
\alpha_\psi(s) &= -2.17 + 0.33\ s \quad .
\end{aligned} \tag{5.4}
$$

On the other hand, if one assumes[23] a universal slope for all bosons, the trajectories are

$$
\begin{aligned}
\alpha_\rho(s) &= 0.47 + 0.88\ s \\
\alpha_{K^*}(s) &= 0.30 + 0.88\ s \\
\alpha_\phi(s) &= 0.08 + 0.88\ s \\
\alpha_{D^*}(s) &= -2.56 + 0.88\ s \\
\alpha_\psi(s) &= -7.45 + 0.88\ s \quad .
\end{aligned} \tag{5.5}
$$

In Table I we give the predicted masses of resonances on the ρ, K^*, ϕ, D^*, ψ trajectories respectively for the new scheme and for the universal-slope scheme.

B. Baryons

We give here predictions for the baryon slopes. The use of Eqs. (4.3), (4.7), (4.8), (4.9), (4.10), (4.11) together with the boson slopes given in Eqs. (5.1), (5.2), (5.3) immediately leads us to

$$
\begin{aligned}
\alpha_{\Delta}' &= 0.88 \\
\alpha_{Y^*}' &= 0.83^{24} \\
\alpha_{\Xi^*}' &= 0.79 \\
\alpha_{C^*}' &= 0.54 \\
\alpha_{S^*}' &= 0.51 \\
\alpha_{X^*}' &= 0.33 \quad .
\end{aligned}
\tag{5.6}
$$

The other baryon slopes, which are not directly related to boson slopes, are calculated, using Eqs. (3.14) ~ (3.17), to be

$$
\begin{aligned}
\alpha_{\Omega}' &= 0.75 \\
\alpha_{X_s^*}' &= 0.31 \\
\alpha_{T^*}' &= 0.48 \\
\alpha_{\Theta}' &= 0.20 \quad .
\end{aligned}
\tag{5.7}
$$

(In the case of $\alpha_{\psi}' = 0.5$ (Ref. 20), see Ref. 25.)

If we connect $\Delta(7/2^+)$ and $\Delta(3/2^+)$ to form the Δ trajectory, we obtain $\alpha_\Delta{}' = 0.88$, which agrees with the predicted value in Eq. (5.6). If we connect $Y_1^*(7/2^+)$ and $Y_1^*(3/2^+)$ to form the Y_1^*, we have $\alpha_{Y_1^*}{}' = 0.91$. If we connect $Y_1^*(5/2^-)$ and $Y_1^*(3/2^+)$, however, we obtain $\alpha_{Y_1^*}{}' = 0.83$, which agrees with the predicted value. More precise experiments are needed here.

We are now in a position to discuss the spectroscopy of the new baryons. Using the above baryon slopes and masses,[21] we have the baryon trajectories:

$$\begin{aligned}
\alpha_\Delta(s) &= 0.16 + 0.88\ s \\
\alpha_{Y^*}(s) &= -0.09 + 0.83\ s \\
\alpha_{\Xi^*}(s) &= -0.35 + 0.79\ s \\
\alpha_\Omega(s) &= -0.60 + 0.75\ s \\
\alpha_{C^*}(s) &= -1.88 + 0.54\ s \quad .
\end{aligned} \tag{5.8}$$

In Table II, we list the predicted masses of resonances on the Δ, Y_1^*, Ξ^*, Ω, C^* trajectories respectively for the new scheme and for the universal-slope scheme.

VI. CONCLUDING REMARKS AND DISCUSSIONS

In this paper, we discussed in detail a factorization property of Regge slopes between ordinary and new hadrons proposed earlier.[11,12]

The Eq. (2.16) relating slopes between ordinary and strange bosons appears to be well supported experimentally. The other relations (2.7) and (2.15) for charmed bosons are difficult to test directly at present. As is seen in Table I, however, the (tensor) D^{**} mass in the new scheme is predicted to be 2.43 GeV for case i) and 2.36 GeV for case ii). On the other hand, the universal-slope assumption for the D^* trajectory predicts the D^{**} mass to be 2.28 GeV. If a resonance were to be found experimentally around (2.40 ± 0.04) GeV for the $D\pi$, $D\pi\pi$, $D\pi\pi\pi$ system, it would be further evidence in support of the slope factorization property. The SPEAR experiments could clarify this point. According to the new scheme, the $D^{***}(3^-)$ would be found around (2.72 ± 0.06) GeV rather than at 2.51 GeV, as obtained from the universal-slope hypothesis. The SPEAR experiments could also clarify this point. As can be seen from Table II, the $C_1^*(5/2^-)$ (charmed baryon) is predicted to be (2.82 ± 0.03) GeV in the new scheme.

In conclusion, we emphasize that the accommodation of charm in the duality scheme leads to many interesting predictions.

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Table I. Predicted masses of resonances on the ρ, K^*, ϕ, D^*, ψ trajectories for the new scheme and for the universal-slope scheme.

		Mass (GeV)				
	J^P	$\rho - A_2$	$K^* - K^{**}$	$\phi - f'$	$D^* - D^{**}$	$\psi - \chi$
case i) New scheme ($\alpha_\psi' = 0.33$)	1^-	0.776	0.892	1.02	2.01	3.10
	2^+	1.32	1.41	1.52	2.43	3.56
	3^-	1.70	1.79	1.89	2.78	3.96
case ii) New scheme ($\alpha_\psi' = 0.50$)	1^-	0.776	0.892	1.02	2.01	3.10
	2^+	1.32	1.41	1.52	2.36	3.41
	3^-	1.70	1.79	1.89	2.66	3.69
case iii) Universal-slope scheme ($\alpha' = 0.88$)	1^-	0.776	0.892	1.02	2.01	3.10
	2^+	1.32	1.39	1.48	2.28	3.28
	3^-	1.70	1.75	1.82	2.51	3.45

Table II. Predicted masses of resonances on the Δ, Y_1^*, Ξ^*, Ω, C^* trajectories for the new scheme and for the universal-slope scheme.

	J^P	Δ	Y_1^*	Ξ^*	Ω	C_1^*
			Mass (GeV)			
case i) New scheme ($\alpha_{X^*}' = 0.33 = \alpha_\psi'$)	$3/2^+$	1.23	1.38	1.53	1.67	2.50
	$5/2^-$		1.77	1.90		2.85
	$7/2^+$	1.95	2.08	2.21	2.34	3.16
case ii) New scheme ($\alpha_{X^*}' = 0.50 = \alpha_\psi'$)	$3/2^+$	1.23	1.38	1.53	1.67	2.50
	$5/2^-$		1.77	1.90		2.79
	$7/2^+$	1.95	2.08	2.21	2.34	3.05
case iii) Universal-slope scheme ($\alpha' = 0.88$)	$3/2^+$	1.23	1.38	1.53	1.67	2.50
	$5/2^-$		1.75	1.87		2.72
	$7/2^+$	1.95	2.05	2.15	2.25	2.92

REFERENCES

[1] G. Goldhaber et al., Phys. Rev. Lett. 37 (1976) 255; I. Peruzzi et al., Phys. Rev. Lett. 37 (1976) 569; B. Knapp et al., Phys. Rev. Lett. 37 (1976) 882.

[2] S. L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D1 (1970) 1285.

[3] M. K. Gaillard, B. W. Lee and J. L. Rosner, Rev. Mod. Phys. 47 (1975) 277.

[4] For a review, see M. Fukugita and K. Igi. University of Tokyo preprint, UT-260, Phys. Reports (to be published).

[5] S. Mandelstam, Phys. Rev. Lett. 21, 1724 (1968).

[6] K. Igi, BNL preprint, August, 1976; Phys. Rev. (to be published).

[7] C. G. Callan et al., Phys. Rev. Lett. 34, 52 (1975).

[8] J. S. Kang and H. J. Schnitzer, Phys. Rev. D12, 841 (1975).

[9] Y. Nambu, lectures prepared for the Copenhagen Summer Symposium, 1970 (unpublished); Phys. Rev. D10, 4262 (1974); L. Susskind, Nuovo Cimento 69A, 457 (1970); P. Goddard et al., Nucl. Phys. B56, 109 (1973).

[10] M. Gell-Mann, Phys. Rev. Lett. 8, 263 (1962); V. N. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Lett. 8, 343 (1962).

[11] K. Igi, Factorization of Regge Slopes for Ordinary and New Hadrons, FERMILAB-Pub-76/82-THY, October, 1976; Phys. Lett. (to be published).

[12]J. Pasupathy, Phys. Rev. Lett. 37, 1336 (1976). This paper also derived independently the factorization property of boson slopes. In making predictions it assumes, however, that the leading trajectories in all $q_i\bar{q}_j$ sectors pass through the same point independent of flavor. According to this assumption, the masses of the $c\bar{c}$ states with 2^{++}, 1^{++}, 0^{++} are predicted to be 3.71 GeV and the mass of the $c\bar{c}$ state with 1^{--}, corresponding to ψ' (3684), is predicted to be 4.23 GeV. These are in gross disagreement with experiment.

[13]Rigorously speaking, the Regge residue $\beta_{a\bar{c}\rho\bar{b}d}(t)$ in the conventional Regge pole model is defined by $\mathrm{Im}A_{ab\to cd}(s,t)\xrightarrow[s\to\infty]{}\beta_{a\bar{c}\rho\bar{b}d}(t)s^{\alpha_\rho(t)}$. As discussed in Sec. I, however, we assumed that all the boson trajectories (ρ - f, ϕ - f', D^* - D^{**}, ψ - χ) are approximately linear up to moderately high energies, s_A (see Fig. 1). Eqs. (2.3), (2.4) and (2.5) were derived in these energy regions. Thus, the factorization property derived here should be considered to be only approximate. Since the Stirling formula already gives a good approximation for moderately large values of s, we consider the $s\to\infty$ limit hereafter only for convenience. Similar discussions also apply to baryon trajectories.

[14]In $\pi^+\pi^-$ scattering one can reduce the expression (2.8) to a crossing symmetric form in s and t by adding terms which give non-leading behavior in s.

[15]E. L. Berger and G. C. Fox, Phys. Rev. 188, 2020 (1969).

[16] The notation is from Ref. 3 and B. W. Lee, C. Quigg and J. L. Rosner, FERMILAB-Pub, August, 1976.

[17] The author is thankful to Professor H. Abarbanel for discussions on this point.

[18] K. Igi, Phys. Lett. 28B, 330 (1968).

[19] In the framework of SU(2) we require exchange degeneracy only for the Δ. The requirement of exchange degeneracy for other baryons which include a strange (charm) quark is equivalent to requiring SU(3)(SU(4)). The author is thankful to Dr. T. Eguchi for clarifying this point.

[20] In previous papers,[6, 11] we assumed the ψ' (3684) to be the second daughter of the leading ψ with $\alpha_\psi(s) = 3$ in order to obtain the ψ slope, $\alpha_\psi' = 0.5$. Since the $\chi(2^{++})$, which is exchange degenerate with $\psi(1^{--})$, appears to have mass 3550 MeV/c, however, we connect these two resonances in order to obtain the slope of the ψ trajectory in the text.

[21] Particle Data Group, Rev. Mod. Phys. 48, No. 2, Part II (1976).

[22] In a previous paper (Ref. 11) we used $\alpha_\psi' = 0.5$ (see Ref. 20); hence we deduced $\alpha_{D^*}' = 0.66$. Using $M(D^*) = 2.01$ GeV, we predicted the D^{**} mass to be 2.36 GeV. We also obtained $\alpha_{F^*}' = 0.63$. The Regge trajectories in this case are $\alpha_\rho(s) = 0.47 + 0.88\ s$, $\alpha_{K^*}(s) = 0.34 + 0.83\ s$, $\alpha_\phi(s) = 0.18 + 0.79\ s$, $\alpha_{D^*} = -1.67 + 0.66\ s$, $\alpha_\psi(s) = -3.80 + 0.50\ s$.

[23] J. Finkelstein and S. S. Pinsky, The Regge Spectroscopy of Charmed Mesons, COO-1545-198.

[24] In obtaining the value of α_{Y^*}', we used the value $\alpha_{K^*}' = 0.83$ derived from factorization.

[25] In the case of $\alpha_{\psi}' = 0.5$ (Ref. 20), we obtain $\alpha_{\Delta}' = 0.88$, $\alpha_{Y^*}' = 0.83$, $\alpha_{\Xi^*}' = 0.79$, $\alpha_{C^*}' = 0.66$, $\alpha_{S^*}' = 0.63$, $\alpha_{X^*}' = 0.50$, $\alpha_{\Omega}' = 0.75$, $\alpha_{X_a^*}' = 0.48$, $\alpha_{T^*}' = 0.60$, $\alpha_{\Theta}' = 0.38$. The Regge trajectories in this case are the same as in Eq. (5.8) of the text except for $\alpha_{C^*}(s) = -2.63 + 0.66\ s$.

FIGURE CAPTIONS

Fig. 1: Schematic view of the boson trajectories: ρ - f, ϕ - f', D^* - D^{**} and ψ - χ. All our considerations apply only to the low and mediately-high energy region $s \leq s_A$.

Fig. 2: Reactions ab $\rightarrow$ B $\rightarrow$ ba, ab $\rightarrow$ B $\rightarrow$ dc and cd $\rightarrow$ B $\rightarrow$ dc in the u-channel.

Fig. 3: The Δ, Y^*, Ξ^*, Ω, C^*, S^*, T^*, X^*, X_s^*, Θ trajectories as functions of $M^2 - \left(M_{3/2}^{\ *}\right)^2$.

Fig. 4: The ρ, K^*, ϕ, D^*, F^*, ψ trajectories as functions of $M^2 - M_V^{\ 2}$.

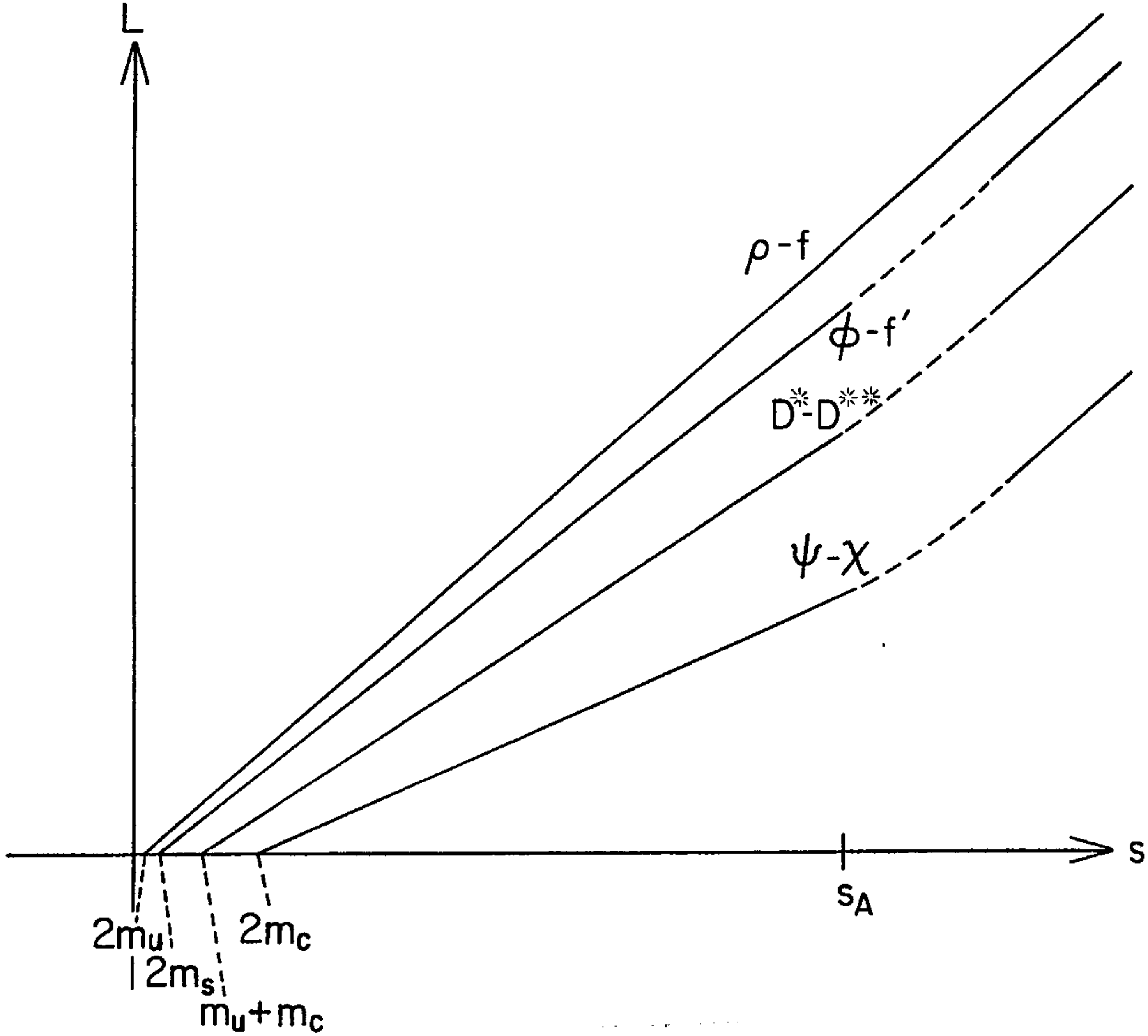

Fig. 1

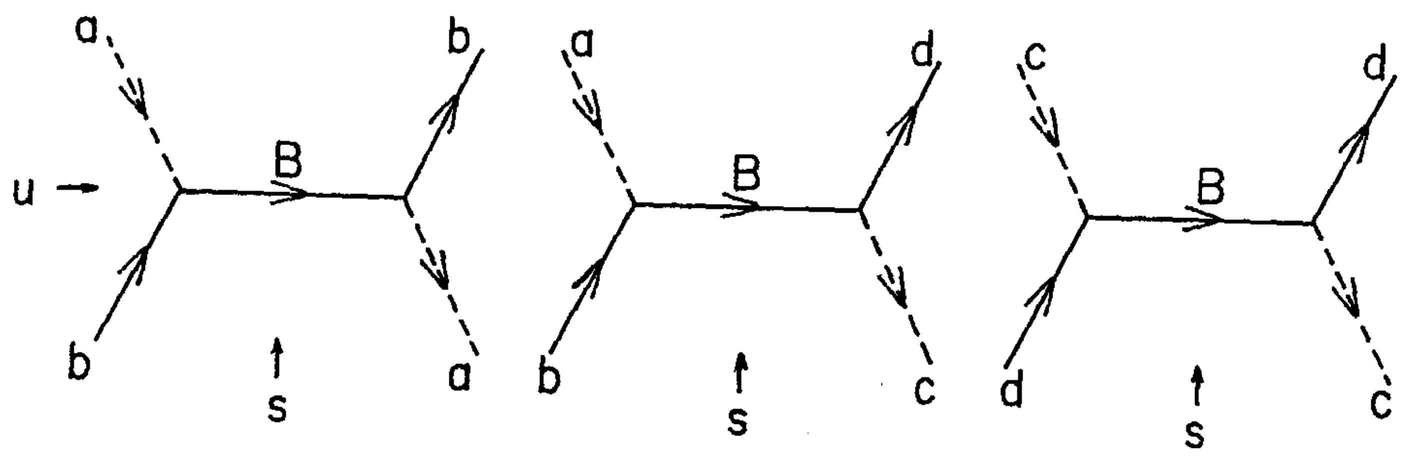

Fig. 2

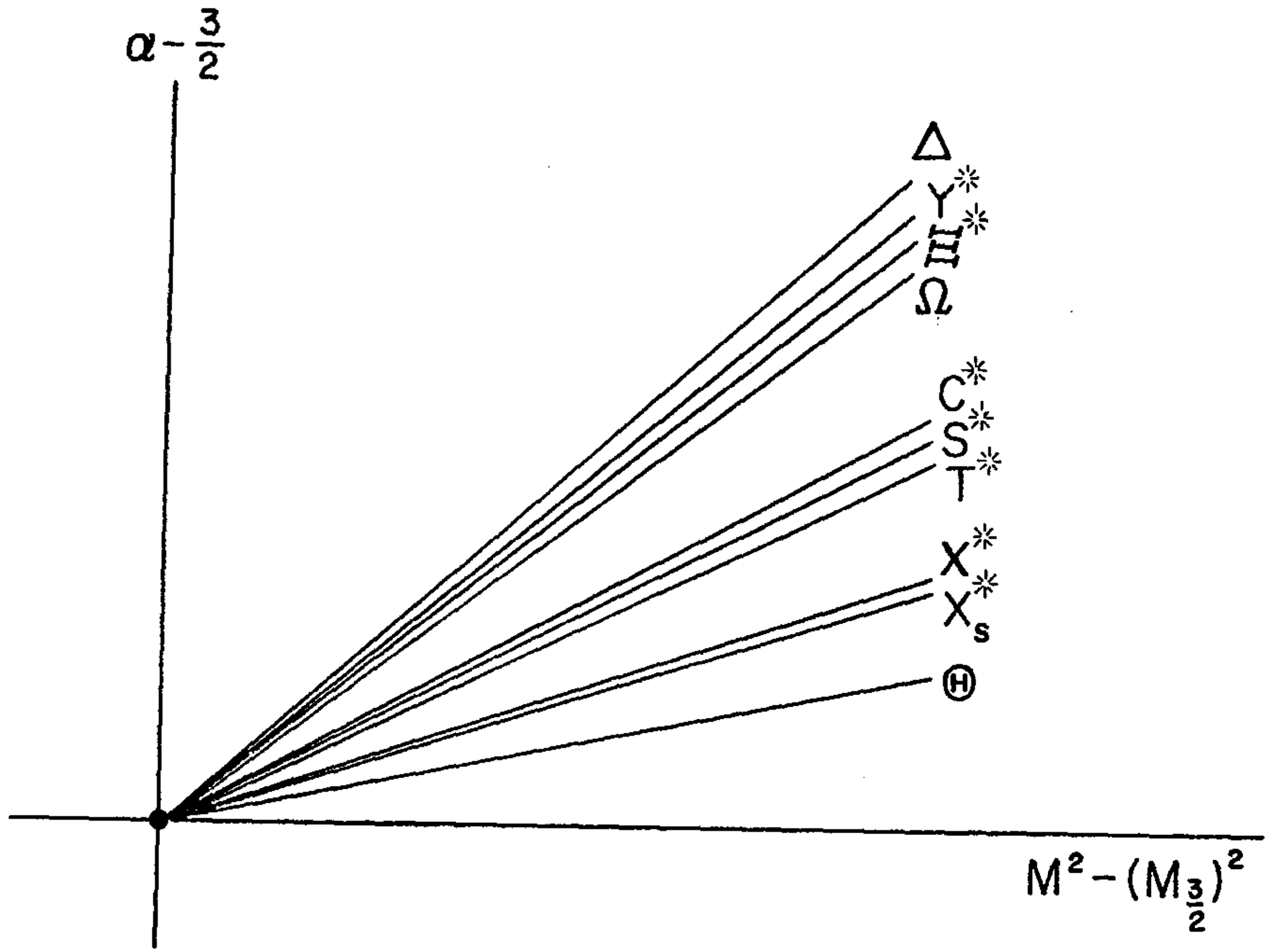

Fig. 3

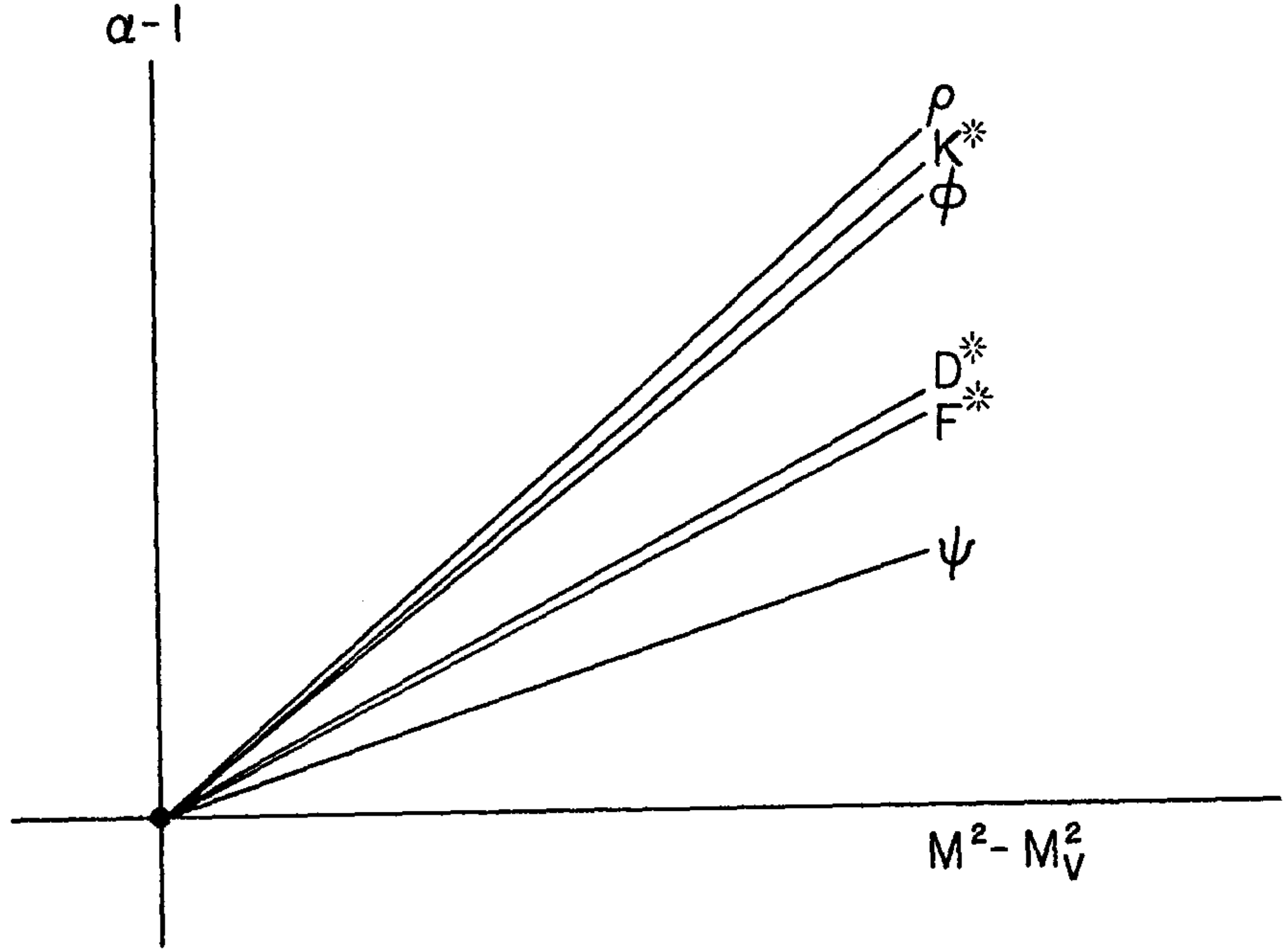

Fig. 4